Subsidies and Corporate Social Responsibility in a Successive Cournot Oligopoly

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Abstract

We consider a successive Cournot oligopoly with competition between vertically integrated and unintegrated firms. Furthermore, we consider the case where it is possible to introduce corporate social responsibility (CSR) into firms' objectives: the firms could be non-profit maximizers. We find that the first-best allocation can be achieved using a uniform per-unit subsidy regardless of firm distribution and the weight of CSR in the firms' objectives.

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1. Introduction

Whether vertical integration impedes competition and worsens social welfare is a major topic in the field of industrial organization. Studies propose contrasting approaches to explore the impact of vertical integration. Greenhut and Ohta (1979) and Salinger (1988) pioneered the field using the successive oligopolies framework\(^1\). These studies indicate that the impact of vertical integration on social welfare depends on the distribution of integrated and unintegrated firms; in this case, a subsidy policy could be effective because oligopolies will continue in the final goods market even if vertical integration improves social welfare. As one objective, this study aims to provide a theoretically distribution of how a uniform subsidization

\(^1\) There are also other types of studies. Ordover et al. (1990) and Chen (2001) use a different model to examine the endogenous decision related to vertical integration besides the aforementioned topic. Of course, there are studies that examine the endogenous decision in the successive Cournot oligopoly framework. See, for example, Abiru et al. (1998) and Buehler and Schmutzler (2008).
policy works in a successive oligopoly wherein vertically integrated and unintegrated firms compete. If the first-best allocation is possible regardless of firm distribution, we can ignore the aforementioned problem by using the subsidy policy.

In 2009, the Japanese government decided to introduce a subsidy for consumers and forwarding agencies who buy new cars that satisfy certain fuel efficiency and emission standards—for example, “eco-cars” such as electric vehicles, plug-in hybrid electric vehicles, and fuel cell vehicles. In the eco-car industry, some intermediate goods producers often function separately from the final goods producers, whereas some are integrated with final goods producers because they have developed products with high technical capabilities, such as for certain consumer electronics goods like laptops and cell phones. This case can serve as an example for our study.

A check of the websites of the firms in the eco-car industry shows that most often emphasize CSR, which includes various activities that do not directly increase profits, for example, regional contribution volunteer activities such as clean-up activities, reconstruction assistance, implementing the traffic safety program, assistance for the arts and education for children, and so on. Considering these activities, firms clearly do not always act as profit maximizers. Therefore, our model allows an analysis of cases where firms are profit maximizers and where they are non-profit maximizers. Matsumura and Ogawa (2014) introduce CSR into firms’ objectives assuming that firms maximize the weighted sum of total social surplus and profit, a setting we adopt. This study’s other aim is to investigate how the weight of CSR in each firm’s objectives affect the equilibrium subsidy level.

This paper is organized as follows. Section 2 describes our model. Sections 3 derives the conditions to realize the first-best allocation, and section 4 considers a non-linear demand function and shows that it is possible to realize the first-best allocation under certain conditions using a uniform per-unit subsidy. Section 5 considers a linear demand function and shows that the aforementioned condition also holds in this case. In addition, we show the results of some comparative statics associated with the weight on CSR. Section 6 concludes the main text.

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2) Regarding battery production, Panasonic and Toyota set up a joint venture, “Panasonic EV Energy Co., Ltd.” while NEC Group and Nissan established “Automotive Energy Supply Corporation”; Toshiba supplies a rechargeable battery, “SCiB,” to the market—for example, to Mitsubishi and Honda.
2. The model

Our model is based on Salinger’s (1988) model. We assume that there are \( M \geq 2 \) upstream firms producing homogeneous intermediate goods and \( N \geq 2 \) downstream firms producing homogeneous final goods. Furthermore, pairs of one upstream firm and one downstream firm may have vertically integrated to form a third type, resulting in three types of firms: unintegrated upstream firms (\( U \) firm), unintegrated downstream firms (\( D \) firm), and vertically integrated firms (\( V \) firm). The number of \( V \) firms is \( n \) such that \( 1 \leq n \leq \min\{M - 1, N - 1\} \).

The profit of \( U \) firm \( k \) can be given by

\[
\pi^U_k = (w - c)x_k, \quad k = 1, \ldots, M - n, \tag{1}
\]

where \( w \) denotes the price of the intermediate good, \( x_k \) the output of \( U \) firm \( k \), \( c \) the marginal production cost, and \( c > 0 \).

Each \( D \) firm uses one intermediate good to produce one final good. \( D \)’s profit is given by

\[
\Pi^D_j = (p - w)q_j, \quad j = n + 1, \ldots, N, \tag{2}
\]

where \( p \) denotes the price of the final good and \( q_j \) the output of \( D \) firm \( j \). Here, the government grants the representative consumer a purchasing subsidy per unit of final good. The inverse demand function is given by \( p = P(Q) + s \), where \( Q \) denotes the total output of final goods and \( s \) denotes the per-unit subsidy level. \( P(Q) \) has the following properties: there exists \( \gamma \in (0, \infty) \) such that \( P(Q) > 0 \) for \( Q \in [0, \gamma) \) and \( P(Q) = 0 \) for \( Q \in [\gamma, \infty) \). \( P(Q) \) is twice-continuously differentiable and \( P'(Q) < 0 \) for all \( Q \) on \( [0, \gamma) \), and \( P(0) > c \).

For \( V \) firms, profit is the joint profit of \( U \) firm and \( D \) firm; that is,

\[
\Pi^V_i = (p - c)q_i, \quad i = 1, \ldots, n, \tag{3}
\]

where \( q_i \) denotes the output of \( V \) firm \( i \). We assume no adjustment costs for firms’ vertical integration and that \( V \) firms do not participate in the intermediate goods market.

Social welfare can now be given by:

\[
W = \left( \int_0^Q P(t)dt - (p - s)Q \right) + \left( \sum_{i=1}^n \Pi^V_i + \sum_{j=n+1}^N \Pi^D_j + \sum_{k=1}^{M-n} \pi^U_k \right) - sQ. \tag{4}
\]

The first curly bracket represents consumer surplus, the second curly bracket represents producer surplus, and the remaining part of \( W \) represents the government’s total subsidy payment.
With respect to each firm’s objective, we follow the setting provided by Matsumura and Ogawa (2014). Each $V$ and $D$ firm aims to maximize the weighted sum of social welfare and its profit:

$$U^V_i = (1 - \theta^V)\Pi^V_i + \theta^V W, \quad i = 1, \cdots, n,$$  

$$U^D_j = (1 - \theta^D)\Pi^D_j + \theta^D W, \quad j = n + 1, \cdots, N,$$  

where $\theta^V, \theta^D \in [0, 1)$ and are exogenously given. On the other hand, each $U$ firm aims to maximize its profit. We use this setting without CSR because it is too complex for analysis.

This is a three-stage game. First, the government chooses the subsidy level. Second, $U$ firms choose their output levels and then determine the price of their intermediate goods. Finally, $D$ and $V$ firms choose their output levels simultaneously.

We analyze this case in a partial equilibrium framework and focus on a symmetric equilibrium for each type of firm.

3. First-best allocation

We now consider the first-best allocation. Under the efficient production of intermediate goods, we transform $W$ into

$$W = \int_0^Q P(t) dt - cQ.$$  

The first-order welfare maximizing condition is

$$\frac{dW}{dQ} = P(Q^{FB}) - c = 0,$$  

where $Q^{FB}$ denotes the first-best total output of final goods. From this result, we find no unique first-best allocation, that is, all allocations that satisfy $Q^{FB} = \sum_i q_i + \sum_j q_j$ can be the first-best allocation.

4. The equilibrium outcome and main results

We now solve the three-stage game using backward induction. First, we consider the third stage, where we assume that the marginal revenue of each firm decreases with the increase in its rivals’ output: $P'(Z + q) + P''(Z + q)q < 0$ for all non-negative $Z$ and $q$ with $Z + q \in [0, \gamma)$. The assumptions mentioned earlier combined with this condition guarantee a unique Nash
equilibrium in the third stage. The first-order conditions of the maximization problems for \( V \) firm \( i \) and \( D \) firm \( j \) are as follows:

\[
\frac{\partial U_i^V}{\partial q_i} = P(Q) - c + (1 - \theta^V)(P'(Q)q_i + s) = 0, \quad i = 1, \ldots, n, \tag{9}
\]

\[
\frac{\partial U_j^D}{\partial q_j} = P(Q) - w + (1 - \theta^D)(P'(Q)q_j + s) = 0, \quad j = n + 1, \ldots, N. \tag{10}
\]

From the above \( n \) number of first-order conditions for \( V \) firms and \( N - n \) number of first-order conditions for \( D \) firms, we obtain the following equilibrium output in the third stage:

\[
q_{sb}^i = q^V(w, s; c, \theta^V, \theta^D, n, N), \quad i = 1, \ldots, n, \tag{11}
\]

\[
q_{sb}^j = q^D(w, s; c, \theta^V, \theta^D, n, N), \quad j = n + 1, \ldots, N, \tag{12}
\]

\[
Q_{sb} = Q_{sb}^V(w, s; c, \theta^V, \theta^D, n, N), \tag{13}
\]

where the superscript \( sb \) denotes the equilibrium outcome in the third stage.

Next, we consider the second stage. From the market clearing condition for intermediate goods \( \sum_j q_{sb}^j = X \), where \( X \) represents \( \sum_k x_k \), we obtain the inverse demand function for intermediate good \( w_{sb} = w_{sb}(X, s; c, \theta^V, \theta^D, n, N) \). Now, the profit of \( U \) firm \( k \) becomes

\[
\pi_k^{Usb} = (w_{sb} - c)x_k, \quad k = 1, \ldots, M - n. \tag{14}
\]

Here, we assume that the second-order profit-maximizing condition for each \( U \) firm is satisfied. The first-order profit-maximizing condition for \( U \) firm \( k \) is

\[
\frac{\partial \pi_k^{Usb}}{\partial x_k} = w_{sb} - c + \frac{\partial w_{sb}}{\partial X}x_k = 0, \quad k = 1, \ldots, M - n. \tag{15}
\]

From the above \( M - n \) number of first-order conditions for \( U \) firms, we obtain the following equilibrium output in the second stage:

\[
x_{SB}^k = x_{SB}^V(s; c, \theta^V, \theta^D, n, M, N), \quad k = 1, \ldots, M - n, \tag{16}
\]

where the superscript \( SB \) denotes the equilibrium outcome in the second stage. We denote \( Q^{SB} \) as \( Q_{sb}^{V}(w_{sb}(X_{SB}^V, s; c, \theta^V, \theta^D, n, N), s; c, \theta^V, \theta^D, n, N), \) where \( X_{SB}^V = \sum_k x_{SB}^k \).

Last, we consider the first stage. The government maximizes social welfare by choosing the appropriate subsidy level. From the balance of revenue and expenditures for the intermediate goods, we rewrite social welfare as

\[\text{See Gaudet and Salant (1991).}\]
\[ W^{SB} = \int_0^{Q^{SB}} P(t) dt - cQ^{SB}. \]  

Here, we assume that the second-order welfare-maximizing condition is satisfied. The first-order welfare-maximizing condition is given by

\[ \frac{\partial W^{SB}}{\partial s} = (P(Q^{SB}) - c) \frac{\partial Q^{SB}}{\partial s} = 0. \]

Now, we obtain the following proposition.

**Proposition 1** Suppose there is a successive Cournot oligopoly where vertically integrated and unintegrated firms with CSR compete. If \( \frac{\partial Q^{SB}}{\partial s} \neq 0 \), the first-best allocation can be achieved by using a uniform per-unit subsidy regardless of firm distribution and the weight of CSR in firms’ objectives.

**Proof** When \( \frac{\partial Q^{SB}}{\partial s} \neq 0 \), the government always chooses a subsidy level that satisfies \( P(Q^{SB}) - c = 0 \) from (18) identical to (8).

In the next section, we consider the case with a linear inverse demand function and find that \( \frac{\partial Q^{SB}}{\partial s} > 0 \) in this case.

The intuition behind Proposition 1 is as follows. From the social welfare perspective, the total output of final goods and production allocation are important. However, production allocation is not important in this case because \( D \) firms’ production cost is perfectly offset by \( U \) firms’ revenue and the marginal cost of producing an intermediate good is identical for \( U \) and \( V \) firms. Only the total output of final goods affects social welfare, and therefore the first-best allocation can be achieved with a uniform subsidy.

5. Linear demand function

In this section, we show that proposition 1 holds when the inverse demand function is linear: \( P(Q) = a - Q \). In the third stage, we solve the maximization problems for each \( V \) and \( D \) firm, where the sum of their first-order conditions are as follows:

\[ n\{a + (1 - \theta^V)s - c\} - (n + 1 - \theta^V)Q^V - nQ^D = 0, \]

\[ (N - n)\{a + (1 - \theta^D)s - w\} - (N - n)Q^V - (N - n + 1 - \theta^D)Q^D = 0, \]

where \( Q^V \equiv \sum q_i \) and \( Q^D \equiv \sum q_j \). To solve the above equations with respect to \( Q^V \) and \( Q^D \), we obtain the following equilibrium total output for \( V \) and \( D \) firms in the third stage.
\[ Q^{V_{sb}} = \frac{n}{\Delta} [(1 - \theta^D)a - (1 - \theta^D + N - n)c + (N - n)w + \{(1 - \theta^D)(1 - \theta^V) + (N - n)(\theta^D - \theta^V)\}s], \]

\[ Q^{D_{sb}} = \frac{N - n}{\Delta} [(1 - \theta^V)a + nc - (n + 1 - \theta^V)w + \{(1 - \theta^D)(1 - \theta^V) - n(\theta^D - \theta^V)\}s], \]

where \( \Delta = (1 - \theta^D)(1 - \theta^V) + N - n\theta^D - (N - n)\theta^V > 0. \)

In the second stage, the price of an intermediate good is determined so as to satisfy \( Q^{D_{sb}} = X, \) and we obtain

\[ w^{sb} = \frac{(N - n)[(1 - \theta^V)a + nc + \{(1 - \theta^D)(1 - \theta^V) - n(\theta^D - \theta^V)\}s] + \Delta X}{(N - n)(n + 1 - \theta^V)}. \]

Using the above inverse demand function of the intermediate good, we solve the maximization problem for \( U \) firms; the sum of their first-order conditions are as follows:

\begin{align*}
(M - n)(an - cn - aN + cN + ns - Ns - ns\theta^D - n^2s\theta^D + Ns\theta^D + nNs\theta^D - an\theta^V + cn\theta^V + aN\theta^V - cN\theta^V - ns\theta^V + n^2s\theta^V + Ns\theta^V - nNs\theta^V + ns\theta^D\theta^V - Ns\theta^D\theta^V) + (M - n + 1)\Delta X = 0. \tag{24}
\end{align*}

To solve the above equation with respect to \( X \) and to derive \( x^{SB} = X^{SB}/(M - n), \) we obtain

\[ x^{SB} = \frac{(N - n)[(a - c)(1 - \theta^V) + s\{(1 - \theta^D)(1 - \theta^V) - n(\theta^D - \theta^V)\}]}{(M - n + 1)\Delta}. \tag{25}
\]

The equilibrium total output \( Q^{SB} \) in the second stage is:

\[ Q^{SB} = \frac{1}{(M - n + 1)(n + 1 - \theta^V)\Delta} \{ (a - c)(n + MN + MnN - n^2N - n\theta^D) - Mn\theta^D - Mn^2\theta^D + n^3\theta^D - n^2\theta^V + Mn\theta^V + Mn^2\theta^V - n^2\theta^V - 2MN\theta^V + nN\theta^V - MnN\theta^V + n^2N\theta^V + n\theta^D\theta^V + Mn\theta^D\theta^V - n^2\theta^D\theta^V - Mn(\theta^V)^2 + n^2(\theta^V)^2 + MN(\theta^V)^2 - nN(\theta^V)^2 + (1 - \theta^V)\Delta^* s \}, \tag{26}
\]

where \( \Delta^* = n + MN + MnN - n^2N - n\theta^D - n^2\theta^D - MN\theta^D + nN\theta^D - MnN\theta^D + n^2N\theta^D - n\theta^V + n^2\theta^V - Mn\theta^V + n\theta^D\theta^V + MN\theta^D\theta^V - nN\theta^D\theta^V > 0. \) We find that \( \partial Q^{SB}/\partial s > 0. \)

In the first stage, the government maximizes social welfare in terms of \( s. \) To solve the first-order condition, we obtain

\[ s^* = \frac{(a - c)\{(M - n + 1)(1 - \theta^D)(n + 1 - \theta^V) + (N - n)(1 - \theta^V)\}}{\Delta^*}. \tag{27}
\]

We then obtain the equilibrium outcome in the full game and show some results below.
\[ W^* = \frac{(a - c)^2}{2}, \quad (28) \]
\[ Q^* = a - c, \quad (29) \]
\[ Q^{D*} = \frac{(a - c)(M - n)(N - n)(1 - \theta^D)(1 + n - \theta^V)}{\Delta^*}, \quad (30) \]
\[ Q^{V*} = ns^*, \quad (31) \]
\[ x^* = \frac{(a - c)(N - n)(1 - \theta^D)(1 + n - \theta^V)}{\Delta^*}, \quad (32) \]

where the asterisk indicates the equilibrium outcome in the full game. We find the maximum social welfare, or the first-best allocation, a uniform per-unit subsidy.

Here, we examine how the weight of CSR in each firm’s objectives affect the equilibrium subsidy level, \( V \) and \( D \) firms’ total output, and obtain the following proposition.

**Proposition 2** When the weight of CSR in the objective of vertically integrated firm \( \theta^V \) (unintegrated downstream firm \( \theta^D \)) increases: 1. the equilibrium subsidy level \( s^* \) decreases (increases), 2. the total output of vertically integrated firm \( Q^V \) decreases (increases), and 3. the total output of unintegrated downstream firm \( Q^D \) increases (decreases).

**Proof** The following summarizes the comparative statics.

\[
\frac{\partial s^*}{\partial \theta^V} = -\frac{(a - c)n(M - n)(N - n)^2(1 - \theta^D)}{(\Delta^*)^2} < 0, \quad (33)
\]
\[
\frac{\partial Q^{V*}}{\partial \theta^V} = -\frac{(a - c)n^2(M - n)(N - n)^2(1 - \theta^D)}{(\Delta^*)^2} < 0, \quad (34)
\]
\[
\frac{\partial Q^{D*}}{\partial \theta^V} = \frac{(a - c)n^2(M - n)(N - n)^2(1 - \theta^D)}{(\Delta^*)^2} > 0, \quad (35)
\]
\[
\frac{\partial s^*}{\partial \theta^D} = \frac{(a - c)(M - n)(N - n)^2(1 + n - \theta^V)(1 - \theta^V)}{(\Delta^*)^2} > 0, \quad (36)
\]
\[
\frac{\partial Q^{V*}}{\partial \theta^D} = \frac{(a - c)n(M - n)(N - n)^2(1 + n - \theta^V)(1 - \theta^V)}{(\Delta^*)^2} > 0, \quad (37)
\]
\[
\frac{\partial Q^{D*}}{\partial \theta^D} = -\frac{(a - c)n(M - n)(N - n)^2(1 + n - \theta^V)(1 - \theta^V)}{(\Delta^*)^2} < 0, \quad (38)
\]

The intuition behind Proposition 2 is difficult to explain. For example, we pick up the case of \( \frac{\partial s^*}{\partial \theta^V} < 0 \) and \( \frac{\partial s^*}{\partial \theta^D} > 0 \). To see their results, we consider the following intuition — when \( \theta^V \) increases, the total output of a vertically integrated firm increases as \( V \) firms produce more to enhance welfare. Their marginal costs are less than those of \( D \) firms, and the total output increases significantly. Therefore, the case requires a lower subsidy. The opposite results hold when \( \theta^D \) increases; however, this is not correct. From (9), an increase in
\(\theta^V\) leads to both a decrease in market power but also a decrease in the subsidy’s importance in the \(V\) firm’s objectives. The former implies that \(V\) firms produce more, while the latter implies that it produces less. Therefore, whether \(\theta^V\) increases the total output of a vertically integrated firm or not depends on the magnitude of the relationships between the above two effects. Unfortunately, this is not yet clear. At this point, we will continue with a deeper analysis using other settings to obtain the correct intuition behind the results. It is interesting that an increase in the weight \(\theta^V\) \((\theta^D)\) decreases the total output of \(V\) \((D)\) firms \(Q^V\) \((Q^D)\); where there is no subsidy, \(V\) \((D)\) firms produce more with an increase in \(\theta^V\) \((\theta^D)\), and thus \(Q^V\) \((Q^D)\) increases as long as the output is positive in the equilibrium. The results would be counterintuitive.

The case where \(\theta^V = \theta^D = 0\): All firms are profit maximizers

Here we consider the special case where \(\theta^V = \theta^D = 0\), that is, all firms are profit maximizer, as in a traditional case. In this case, for some equilibrium outcome, we show that

\[
q_i^{sb} = \frac{a - (N - n + 1)c + s + (N - n)w}{N + 1}, \quad q_j^{sb} = \frac{a + nc + s - (n + 1)w}{N + 1},
\]

\[
Q^{sb} = \frac{(a + s)N - (N - n)w - nc}{N + 1}, \quad w^{sb} = \frac{(a + s)(N - n) + cn(N - n) - (n + 1)X}{(n + 1)(N - n)},
\]

\[
X^{SB} = \frac{(M - n)(N - n)(a - c + s)}{(M - n + 1)(N + 1)}, \quad Q^{SB} = \frac{(a - c + s)\{n + MN + (M - n)nN\}}{(n + 1)(M - n + 1)(N + 1)}.
\]

From the above results, we find that \(\partial Q^{SB}/\partial s > 0\).

Note that the equilibrium subsidy level \(s^*\) is

\[
s^* = \frac{(1 + nM + M + N - n - n^2)(a - c)}{n + MN + nMN - n^2N}
\]

and that it has the following properties: 1. \(\partial s^*/\partial N < 0\) and \(\partial s^*/\partial M < 0\), and 2. the sign of \(\partial s^*/\partial n\) corresponds to the sign of \(2nN - n^2 - M - MN - 1\). For property 1, an increase in \(N\) or \(M\) leads to an increase in \(Q^{SB}\) because it becomes more competitive in the final goods market and therefore the case requires a lower subsidy. Property 2 has the same condition that determines the sign of \(dP_F/dn\) in Salinger (1988), where \(P_F\) is almost the same as \(P(Q)\) in this study, because the subsidy needed is lower (higher) than that needed when an increase in \(n\) leads to an increase (decrease) in \(Q^{SB}\); that is, a decrease (increase) in \(P(Q)\).

6. Concluding remarks

This paper investigates the effect of a uniform per-unit subsidy in a successive Cournot
oligopoly with competition between vertically integrated and unintegrated downstream firms with CSR. We show that the first-best allocation can be realized with a uniform subsidy, even when the two types of firms have different objective functions and marginal costs for final goods in the downstream market: the weight of CSR in vertically integrated firm $\theta^V$ could differ from that of unintegrated downstream firm $\theta^D$, and the marginal cost of vertically integrated firm $c$ could be smaller than that of unintegrated downstream firm $w$.

We note that proposition 1 holds even when introducing a new stage where firms can decide whether to integrate into the model. Therefore, we can ignore firms’ incentives for vertical integration under a subsidization policy in terms of social welfare. However, note that this depends on two assumptions: both vertically integrated firms and unintegrated upstream firms have identical constant marginal costs of the intermediate good and firms produce homogeneous goods in each market.

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